Period 4, April 3, 2025

Definition of a Hyperbola

A **hyperbola** is the set of points in a plane the difference of whose distances from two fixed points, called foci, is constant.

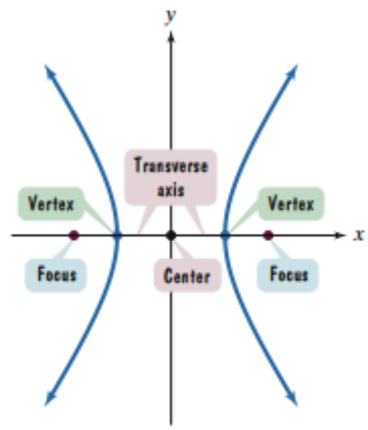


Figure 9.15 The two branches of a hyperbola

Notice the sign difference between the following equations:

Finding an ellipse's foci:

$$c^2 = a^2 - b^2$$

Finding a hyperbola's foci:

$$c^2 = a^2 + b^2.$$

Standard Forms of the Equations of a Hyperbola

The **standard form of the equation of a hyperbola** with center at the origin is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1.$$

Figure 9.17(a) illustrates that for the equation on the left, the transverse axis lies on the x-axis. **Figure 9.17(b)** illustrates that for the equation on the right, the transverse axis lies on the y-axis. The vertices are a units from the center and the foci are c units from the center. For both equations, $b^2 = c^2 - a^2$. Equivalently, $c^2 = a^2 + b^2$.

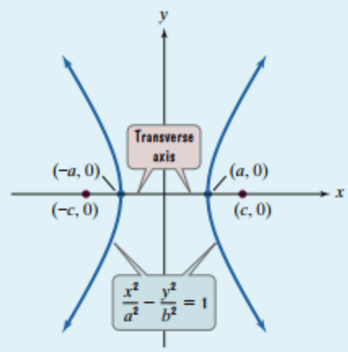


Figure 9.17(a) Transverse axis lies on the x-axis.

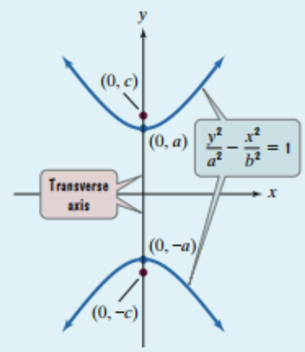


Figure 9.17(b) Transverse axis lies on the y-axis.

$\frac{9}{16} - \frac{y^2}{9} = 1 \Rightarrow \frac{9}{16} - \frac{y^2}{9} = \frac{16}{16} - \frac{9}{16}$

$-\frac{9}{16}$

$-\frac{y^2}{9} = \frac{7}{16} \cdot 9$

$-y^2 = \frac{63}{16}$

$\cancel{y^2 = -\frac{63}{16}}$

given equation.

opens Left/Right

a. $\frac{x^2}{16} - \frac{y^2}{9} = 1$

Center (0,0)

$\sqrt{16} = 4$

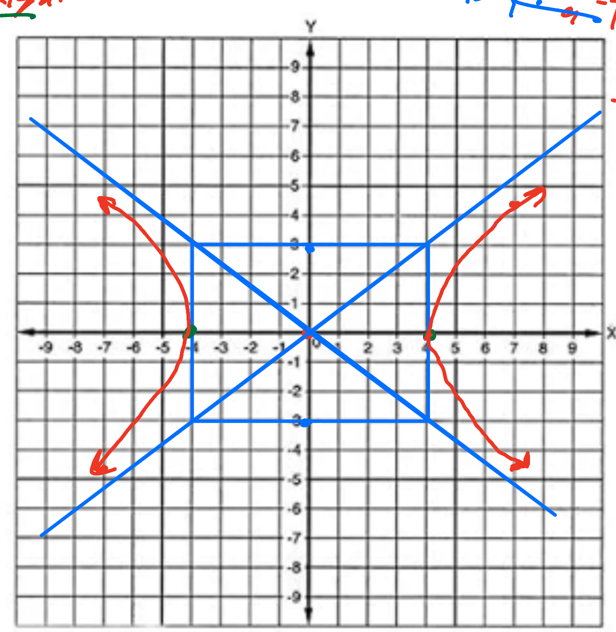
$\sqrt{9} = 3$

$\frac{49}{16} - \frac{y^2}{9} = \frac{16}{16}$

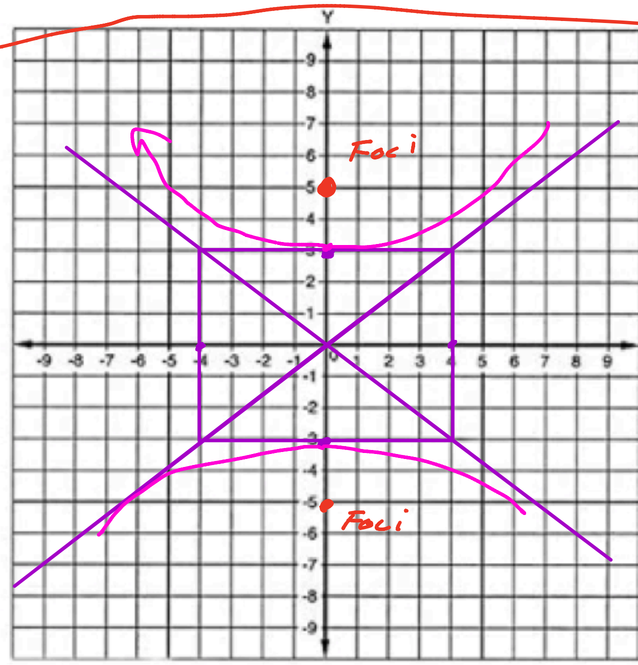
$-\frac{y^2}{9} = -\frac{33}{16}$

$y^2 = \frac{297}{16} \Rightarrow y = \pm 4.308$

x	y
3	∅
±4	0
±7	±4.308



b. $\frac{y^2}{9} - \frac{x^2}{16} = 1.$



Opens UP/Down

Foci^2 = $3^2+4^2 = 9+16$

Foci^2 = 25

Foci = 5

Find the standard form of the equation of a hyperbola with foci at (0, −3) and (0, 3) and vertices (0, −2) and (0, 2), shown in **Figure 9.20**.

Center (0,0)

Focal LengTh=3

VerTices (0,-2) and (0,2)

Opens UP / Down

$3^2 = 2^2 + b^2$

$9 = 4 + b^2$

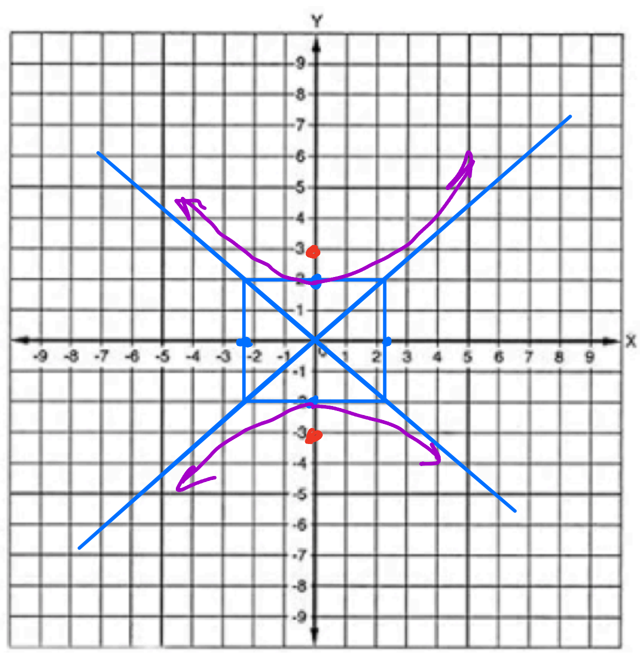
$5 = b^2$

$\sqrt{5} = b = 2.23$

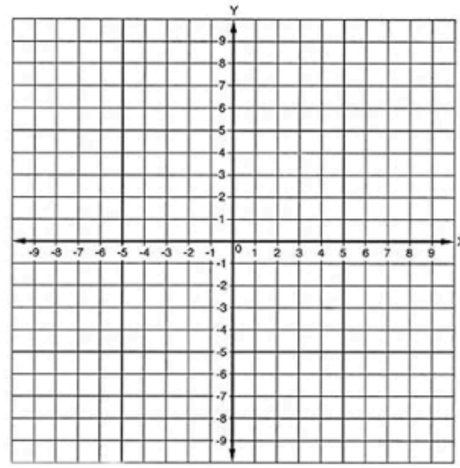
$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$\frac{(y-0)^2}{2^2} - \frac{(x-0)^2}{(\sqrt{5})^2} = 1$

$\frac{y^2}{4} - \frac{x^2}{5} = 1$



CHECK POINT 2 Find the standard form of the equation of a hyperbola with foci at $(0, -5)$ and $(0, 5)$ and vertices $(0, -3)$ and $(0, 3)$.



Graph and locate the foci: $\frac{x^2}{25} - \frac{y^2}{16} = 1$. What are the equations of the asymptotes?

$y = \frac{4}{5}x + 0$

$\sqrt{16} = 4$

Opens Left/Right

Verticies

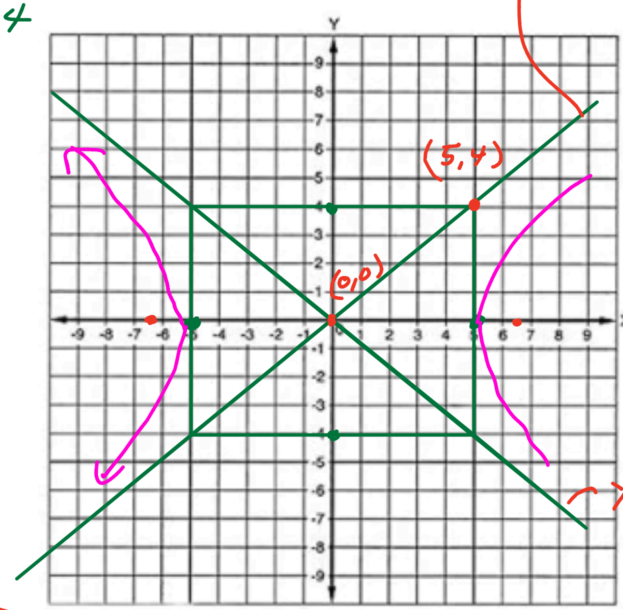
$(\sqrt{25}, 0)$ and $(-\sqrt{25}, 0)$

$(5, 0)$ and $(-5, 0)$

$\text{Foci}^2 = 25 + 16$

$F^2 = 41$

$F = \pm\sqrt{41} = \pm 6.4$



$y = -\frac{4}{5}x + 0$

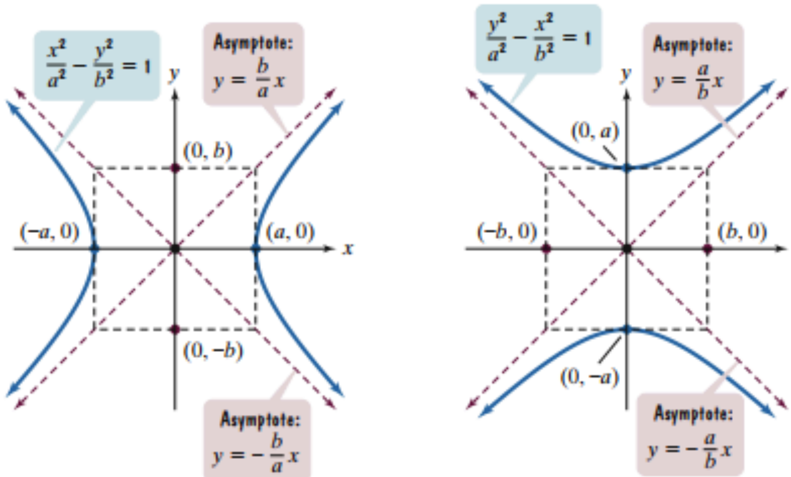


Figure 9.21 Asymptotes of a hyperbola

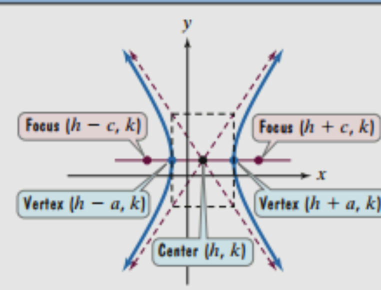
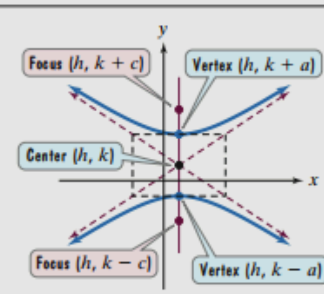
The Asymptotes of a Hyperbola Centered at the Origin

The hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ has a horizontal transverse axis and two asymptotes

$$y = \frac{b}{a}x \quad \text{and} \quad y = -\frac{b}{a}x.$$

The hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ has a vertical transverse axis and two asymptotes

$$y = \frac{a}{b}x \quad \text{and} \quad y = -\frac{a}{b}x.$$

Equation	Center	Transverse Axis	Vertices	Graph
$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ Vertices are a units right and a units left of center. Foci are c units right and c units left of center, where $c^2 = a^2 + b^2$.	(h, k)	Parallel to the x-axis; horizontal	$(h - a, k)$ $(h + a, k)$	
$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ Vertices are a units above and a units below the center. Foci are c units above and c units below the center, where $c^2 = a^2 + b^2$.	(h, k)	Parallel to the y-axis; vertical	$(h, k - a)$ $(h, k + a)$	

r / Right

opens

Graph: $\frac{(x-2)^2}{16} - \frac{(y-3)^2}{9} = 1.$

Center $(2,3)$

$\sqrt{16} = 4$ = Left/Right

$\sqrt{9} = 3$ = up/Down

asymptote $(6,6)$ $(-2,0)$

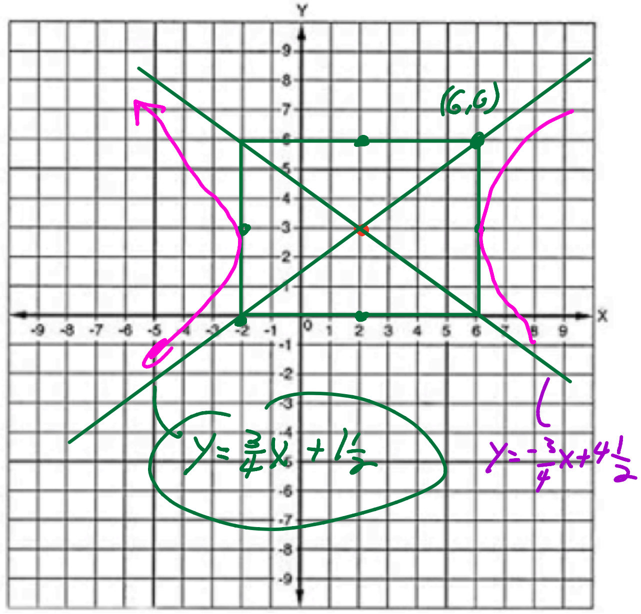
Slope $= \frac{6-0}{6-(-2)} = \frac{6}{8} = \frac{3}{4}$

$y = \frac{3}{4}x + b$

$6 = \frac{3}{4}(6) + b$

$6 = \frac{18}{4} + b \Rightarrow 6 = 4\frac{1}{2} + b$ $\quad b = 1\frac{1}{2}$

$-4\frac{1}{2} \quad -4\frac{1}{2}$



Graph: $4x^2 - 24x - 25y^2 + 250y - 489 = 0$. Where are the foci located? What are the equations of the asymptotes? $+489$ $+489$

$4x^2 - 24x \quad -25y^2 + 250y \quad = 489$

$4(x^2 - 6x + 9) - 25(y^2 - 10y + 25) = 489 + 36 - 625$

$a = 1$ $\quad a = 1$

$b = -6$ $\quad b = -10$

$\frac{b}{2} = \frac{-6}{2} = -3$ $\quad \frac{b}{2} = \frac{-10}{2} = -5$

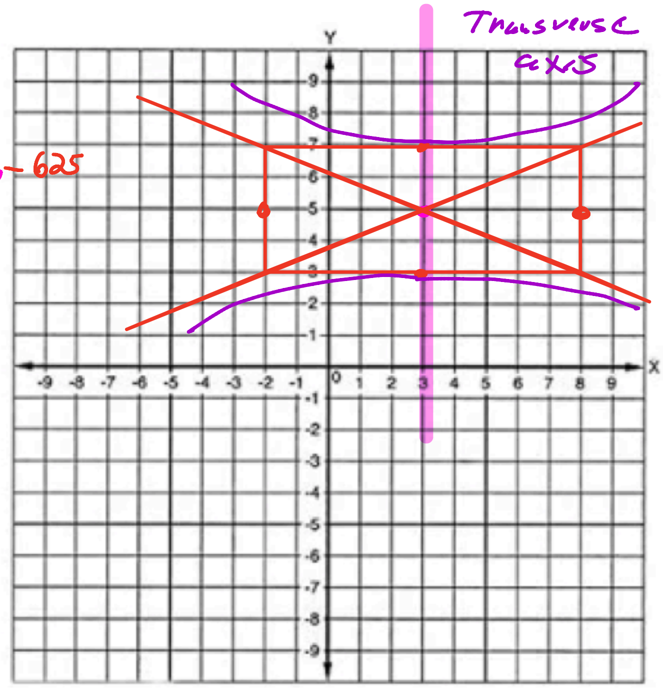
$\left(\frac{b}{2}\right)^2 = (-3)^2 = 9$ $\quad \left(\frac{b}{2}\right)^2 = (-5)^2 = 25$

$4(x-3)^2 - 25(y-5)^2 = -100$

$-100 \quad -100$

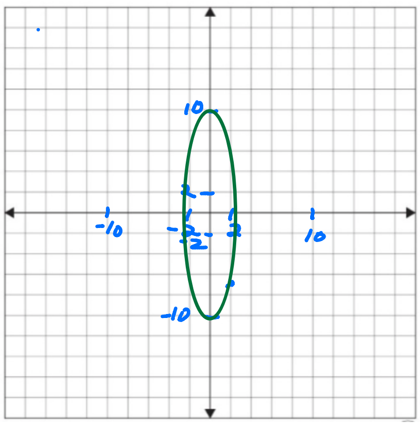
$\frac{4(x-3)^2}{-100} - \frac{25(y-5)^2}{-100} = 1$

$\frac{(y-5)^2}{4} - \frac{(x-3)^2}{25} = 1$ center $(3,5)$



midpoint (0,0)

Find the standard form of the equation of an ellipse with vertices at (0, − 10) and (0,10), passing through (1, − 7).



center (0,0)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{x^2}{a^2}+\frac{y^2}{10^2}=1$$

$$\frac{1^2}{a^2}+\frac{(-7)^2}{100}=\frac{100}{100}$$

$$\frac{1}{a^2}+\cancel{\frac{49}{100}}=\frac{100}{100}-\frac{49}{100}$$

$$-\frac{49}{100}$$

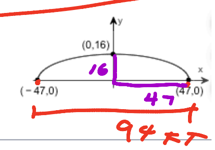
$$\frac{1}{a^2}=\frac{51}{100}$$

$$a^2=\frac{100}{51}$$

$$\frac{x^2}{\frac{100}{51}}+\frac{y^2}{100}=1$$

$$\frac{51x^2}{100}+\frac{y^2}{100}=1$$

The elliptical ceiling of a building is 94 ft long and 16 ft tall. Use the rectangular coordinate system in the figure shown to write the standard form of the equation for the elliptical ceiling. A man discovered that he could hear conversations of colleagues in the entire room if he stood at the focus, (c,0), where $c^2 = a^2 - b^2$. How far along the major axis did the man stand to hear the conversations?



(a) The standard form of the equation is $\square$ = 1.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{x^2}{47^2}+\frac{y^2}{16^2}=1$$

if center (0,0)

$$c^2=a^2-b^2$$

$$c^2=47^2-16^2$$

$$c^2=1953 \qquad c=\sqrt{1953}$$

room if he stood at the focus, (c,0),

$(\sqrt{1953},0)$

44.192

(44,0)

Find (if possible) **a.** AB and **b.** BA

$$A = \begin{bmatrix} 7 & 7 \\ 9 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 6 \\ 4 & -3 \end{bmatrix}$$

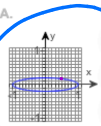
$$\begin{bmatrix} 7 & 7 \\ 9 & 9 \end{bmatrix}\begin{bmatrix} 4 & 6 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 7\cdot4+7\cdot4 & 6\cdot7+7\cdot-3 \\ 9\cdot4+9\cdot4 & 9\cdot6+9\cdot-3 \end{bmatrix}$$

Graph the ellipse and locate the foci.

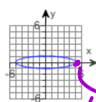
$x^2 = 1 - 25y^2$

Choose the correct graph below.

A.

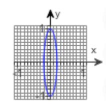


B.

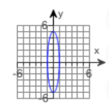


(0,5)

C.



D.



Locate the foci.

(Type ordered pairs. Use a comma to separate answers. Type exact answers, using radicals as needed. Use integers or fractions for any numbers in the expression. Simplify your answers.)

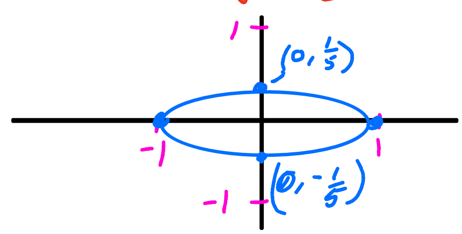
$x^2 = 1 - 25y^2$

$+25y^2 \quad +\cancel{25y^2}$

$x^2 + 25y^2 = 1 \Rightarrow \frac{x^2}{1^2} + \frac{y^2}{\frac{1}{25}} = 1$

Semi-major = 1 → x-axis

Semi-minor = $\sqrt{\frac{1}{25}} = \frac{1}{5}$ → y-axis



$1^2 - \frac{1}{25} = c^2$

$\frac{25}{25} - \frac{1}{25} = c^2$

$\sqrt{\frac{24}{25}} = \sqrt{c^2}$

$c = \frac{\sqrt{24}}{5} = \frac{2\sqrt{6}}{5}$

Foci

$\left(\frac{2\sqrt{6}}{5}, 0\right), \left(-\frac{2\sqrt{6}}{5}, 0\right)$

Find the standard form of the equation of the ellipse satisfying the given conditions.

Foci: $(-7, 0)$, $(7,0)$; vertices: $(-11, 0)$, $(11, 0)$

Type the standard form of the equation.

$$\frac{x}{11^2} + \frac{y^2}{(\sqrt{72})^2} = 1$$

$\frac{x^2}{121} + \frac{y^2}{72} = 1$ (Type an equation. Simplify your answer.)

Foci $(7,0)(-7,0)$

Semiminor

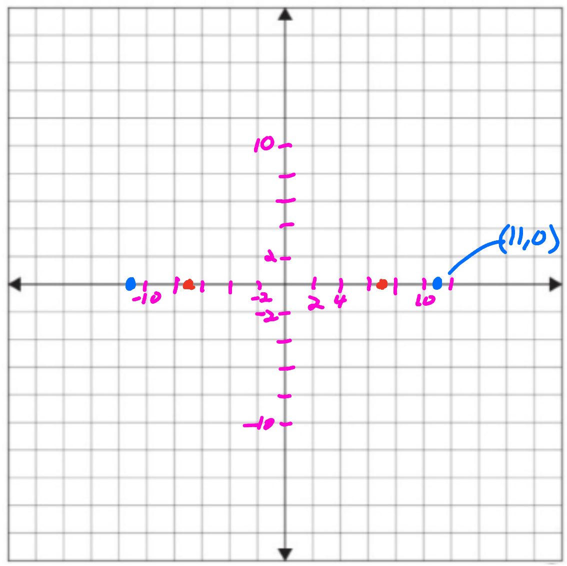
$7^2 + \text{seminimor}^2 = 11^2$

$\cancel{49} + \text{seminimor}^2 = 121$

$-\cancel{49} \qquad -49$

$\sqrt{\text{seminimor}^2} = \sqrt{72}$

$\text{Seminimor} = 6\sqrt{2}$



Find the standard form of the equation of the ellipse satisfying the given conditions.

Endpoints of major axis: (5,5) and (5, − 7)

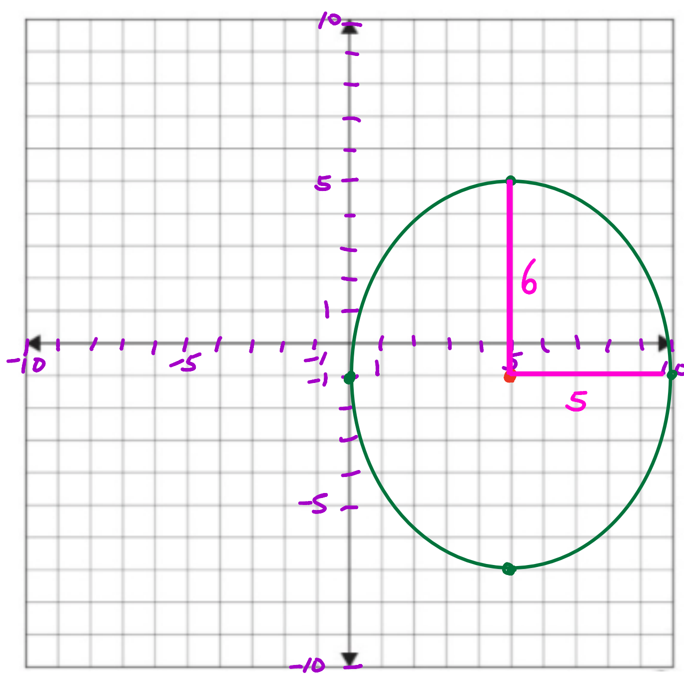
Endpoints of minor axis: (10, − 1) and (0, − 1)

center midpT between (5,5) and (5,−7) or (10,−1)(0,−1) (5,−1) center

Standard form of the equation: $\frac{(x-5)^2}{25}+\frac{(y+1)^2}{36}=1$

$\frac{(x-5)^2}{a^2}+\frac{(y-(-1))^2}{b^2}=1$

$\frac{(x-5)^2}{5^2}+\frac{(y+1)^2}{6^2}=1$



Graph the ellipse and give the location of its foci.

$\frac{(x+2)^2}{25}+(y-2)^2=1$ $=$ $\frac{(x+2)^2}{25}+\frac{(y-2)^2}{1}=1$

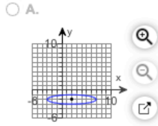
center (−2,2)

Semi major $=\sqrt{25}=5 \Rightarrow$ x−axis LefT/RighT

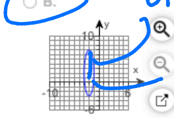
Semi minor $=\sqrt{1}=1 \Rightarrow$ y−axis UP/Down /

Choose the correct graph below.

A.

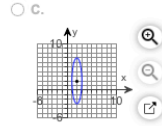


B. UP 5

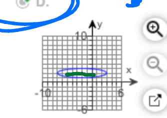


LefT/RighT /

C.



D. LefT/RighT 5



UP /

The foci are located at $(-2-2\sqrt{6},2),(-2+2\sqrt{6},2)$.

(Type ordered pairs. Use a comma to separate answers as needed. Simplify your answers. Type an exact answer, using radicals as needed.)

$5^2=F^2+1^2$

$25=F^2+1$

$24=F^2$

$F=\sqrt{24}=2\sqrt{6}$

center (−2,2)

$(-2+2\sqrt{6},2)$ and $(-2-2\sqrt{6},2)$

Find the standard form of the equation of the ellipse satisfying the given conditions.

Foci: (0, − 7), (0,7); x-intercepts: − 8 and 8 always on major axis

Foci (0,-7) (0,7) Center (0,0)

midpt

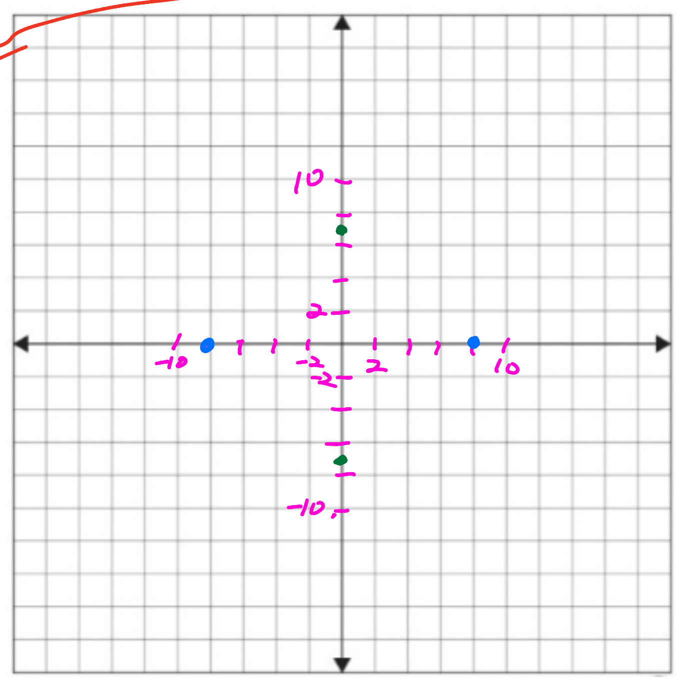
$7^2 + 8^2 = \text{semimajor}^2$

$49 + 64 = 113 = \text{semimajor}^2$

Type the standard form of the equation.

$$\frac{x^2}{64} + \frac{y^2}{113} = 1$$ (Type an equation. Simplify your answer.)

$$\frac{(x-0)^2}{8^2} + \frac{(y-0)^2}{(\sqrt{113})^2} = 1$$



Convert the equation to standard form by completing the square on x and y. Then graph the ellipse and give the location of its foci.

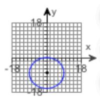
$81x^2+64y^2-1296x=0$

The standard form of the equation is $\frac{(x-8)^2}{64}+\frac{y^2}{81}=1$.

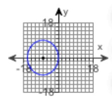
(Type an equation. Simplify your answer.)

Choose the correct graph below.

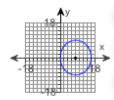
A.



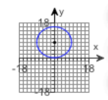
B.



C.



D.



The foci are located at $(8,\sqrt{17}),(8,-\sqrt{17})$.

(Simplify your answers. Type ordered pairs. Use a comma to separate answers as needed. Type an exact answer, using radicals as needed.)

center $(8,0)$

semi major $=\sqrt{81}=9 \Rightarrow y$

semiminor $=\sqrt{64}=8 \Rightarrow x$

$81x^2+64y^2-1296x=0$

$81x^2-1296x \quad + 64y^2 = 0$

$81(x^2-16x+64)+64y^2=0+64\cdot 81 \Rightarrow \frac{81(x-8)^2+64y^2}{5184} = \frac{5184}{5184}$

$a=1$

$b=-16$

$\frac{b}{a}=\frac{-16}{2}=-8$

$\left(\frac{b}{2}\right)^2=(-8)^2=64$

$\frac{81(x-8)^2}{5184}+\frac{64y^2}{5184}=1$

$\frac{\cancel{81}(x-8)^2}{64\cdot\cancel{81}}+\frac{\cancel{64}y^2}{\cancel{64}\cdot 81}=1$

$\frac{(x-8)^2}{64}+\frac{y^2}{81}=1$

$8^2+F^2=9^2$

$64+F^2=81$

$F^2=17$

$F=\sqrt{17}$ $\Rightarrow$ Foci $(8,0+\sqrt{17})$ and $(8,0-\sqrt{17})$